

ALGEBRAIC TOPOLOGY MID-SEMESTRAL EXAMINATION

Attempt all questions. You do not have to prove anything from scratch and you may quote any result proved in class. Total Marks - 20

Question 1 What is the fundamental group of a Klein bottle? Justify your answer. (2 marks)

Question 2 True or false: The diagonal circle $C := \{(x, y) \in S^1 \times S^1 \mid x = y\}$ is a retract of $S^1 \times S^1$. (3 marks)

Question 3 Describe the real projective plane $\mathbb{R}P^2$ as a CW-complex. (3 marks)

Question 4 Construct a topological space whose fundamental group is $\mathbb{Z}/m\mathbb{Z} \oplus \mathbb{Z}/n\mathbb{Z}$ (for any two positive integers m, n). (3 marks)

Question 5 Let X_n be the topological space obtained from the closed 2-disc D^2 by identifying n distinct points on its boundary (to a single point). What is $\pi_1(X_n)$? (3 marks)

Question 6 Let $S = \{(1/n, 0) \mid n \in \mathbb{N}\} \cup \{(0, 0)\}$ considered as a subset of \mathbb{R}^2 . Join each point of S by a line segment to the point $(0, 1)$, and by another line segment to the point $(0, -1)$. Let X be the space which is the union of all these line segments (as a subspace of \mathbb{R}^2). What is $\pi_1(X)$? (3 marks)

Question 7 What is the fundamental group of the space obtained by removing a finite set of points (say n distinct points for some $n \in \mathbb{N}$) from a torus? from the real projective plane? (3 marks)